MODEL OF FLUIDIZED BED EXPANSION

Benitto MAYRHOFER, Lubomír NEUŽIL and František PROCHÁSKA
Department of Chemical Engineering,
Prague Institute of Chemical Technology, 166 28 Prague 6

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Theoretical relations have been derived for the superficial velocity of fluidization under the creeping flow regime for cubic, monoclinic and tetrahedron configurations of spherical particles. The models proposed have been compared with theoretical models published up to the present and with the power-law correlations recommended. The configuration fitting the best was found to be the tetrahedron one. All configurations examined in this work gave correct trend of the velocity of fluidization *versus* porosity dependence. Experimental data indicate that for the bed porosity ranging between 0·45 and 0·68 our models are better than other so far published. However theoretical models of other authors based on the analysis of the behaviour of a fixed bed give better results.

At present, the most readily available characteristics of a homogeneous fluidized bed is its expansion. Numerous attempts have been made to formulate a theoretical model for this characteristics. The models published in the literature start from the two following principal ideas: From analysis of the flow of a fluid through an fixed bed of material (the so-called inner problem), or, from the study of the flow of a fluid past an elementary particle in the bed (the so-called outer problem). In this study we have adopted the latter approach and confronted its results with experimental data of other authors.

THEORETICAL

Consider a fluidized bed of porosity ε composed of identical spherical particles. A Newtonian incompressible fluid pases through the bed under the creeping flow regime. Individual particles in the bed are fixed, regularly distributed in space, and the velocity field in the vicinity of a particle is not distorbed by the presence of other particles.

The velocity component, u_z, in the direction of the bulk flow of the fluid in the equitorial plane around each particle is given by[1]

$$u_z = \left[1 - \frac{3}{4}\frac{R}{r} - \frac{1}{4}\left(\frac{R}{r}\right)^3\right] V_0 \,. \qquad (1)$$

Let us consider three selected configurations of particles in the bed: A cubic (type *A*), an equilateral rhombic (monoclinic system) (type *B*) an a tetrahedron one (type *C*) under the following limitations:

For the type *B* we shall consider the dense packing at an angle φ (Fig. 1) growing with increasing porosity.

For ε exceeding or equal 0·475, type *B* changes into type *A*. The configurations considered and their changes under the flow regime are quite acceptable from the point of energy considerations.

The mean superficial velocity of the fluid, u, *i.e.* the superficial velocity of fluidization is given by

$$u = \frac{1}{S} \oint_{S_1} u_z \, \mathrm{d}S \,. \tag{2}$$

After substituting from Eq. (*1*) into (*2*) and solving for individual configurations, the result may be written in an analytical form as:

Type *A*:

$$u/V_0 = 1 - 3(R/L) \ln (\sqrt{(2)} + 1) + 2\sqrt{(2)}(R/L)^3 \tag{3}$$

where*)

$$R/L = [3(1 - \varepsilon)/4]^{1/3} \,, \quad \varepsilon \geqq 0{\cdot}475 \,. \tag{4}$$

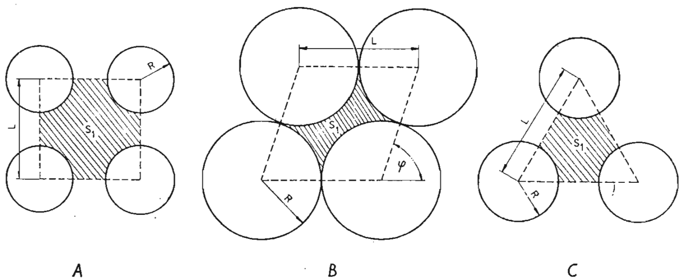

FIG. 1

Elementary cell of Configuration *A*, *B* and *C*

S_1 Free cross section, ——— limits of the total cross sectional area of the elementary cell.

* The lower limit for ε follows from the geometrical arrangement in the model; see also Eqs (*7*) and (*9*) below.

Hence

$$u/V_0 = 1 - 1{\cdot}6406(1 - \varepsilon)^{1/3} + 0{\cdot}6752(1 - \varepsilon)\,. \tag{5}$$

Type *B*:

$$u/V_0 = (\pi/8 \sin \varphi)\,[2/\cos^2(\varphi/2) - 3/\cos(\varphi/2) + \cos(\varphi/2)] + (1/2 \sin \varphi)\,.$$
$$.\left[-\varphi/\cos^2(\varphi/2) + 2\,\mathrm{tg}\,(\varphi/2) + 1{\cdot}5\varphi/\cos(\varphi/2) - 3\ln\frac{1 + \sin(\varphi/2)}{\cos(\varphi/2)} - \right.$$
$$\left. - 0{\cdot}5\varphi\cos(\varphi/2) + \sin(\varphi/2)\right], \tag{6}$$

where

$$(1 - \varepsilon) = \pi/6 \sin^2 \varphi\,, \quad 0{\cdot}26 < \varepsilon \leqq 0{\cdot}475\,. \tag{7}$$

For ε equal 0·475, *i.e.* for $\varphi = \pi/2$, Eq. (*6*) is transformed into Eq. (*4*) with $L = 2R$. For ε exceeding 0·475 the type *B* configuration changes into the type *A* under the assumptions made previously.

Type *C*:

$$u/V_0 = 1 - (R/L)\,1{\cdot}5\sqrt{(3)}\ln 3 + (R/L)^3\,2\sqrt{3} \tag{8}$$

where

$$(1 - \varepsilon) = 5{\cdot}9256(R/L)^3\,, \quad \varepsilon \geqq 0{\cdot}26\,. \tag{9}$$

Hence:

$$u/V_0 = 1 - 1{\cdot}5775(1 - \varepsilon)^{1/3} + 0{\cdot}5846(1 - \varepsilon)\,. \tag{10}$$

DISCUSSION

A plot of V_0/u as a function of bed porosity for individual configurations is shown in Fig. 2 together with data computed from equations published in the literature[2-8]. The curve computed for the type *A* configuration according to Eq. (*4*) is drawn by a dotted line (apart from the section of physical validity of the model). Analytical expressions of the equations taken from the literature which are depicted in Fig. 2 are given in the Appendix. Analysis of experimental data suggests that the most suitable type of expression for the description of the fluidized bed expansion under the laminar regime is the power-law expression

$$u/V_0 = \varepsilon^n\,. \tag{11}$$

The recommended[2,3] value of the exponent n is

$$n = 4{\cdot}65 - 4{\cdot}75\,.$$

Fig. 2 shows the course of Eq. (*11*) for the above limiting values ($n = 4{\cdot}65$ by the dash-and-dot line and for $n = 4{\cdot}75$ by the broken line). Confrontation of the equations published in the literature with the equations of the type (*11*) indicates that the minimum fluidization velocity ($\varepsilon \geqq 0{\cdot}4$) is predicted best by the Ergun equation[8] (inner problem), *i.e.* by curve *3*, by the Happel's equation[7] (outer problem) shown by curve *4*, and by the Kozeny–Carman equation in Todes' modification[2,5,6] (inner problem), shown by curve *6*. Brinkman's equation (curve *5*) predicts too high values of the ratio V_0/u. The tetrahedron configuration is the best zone from the ones (*C*) examined in this work.

A comparison of individual curves provides an interesting result: For the porosity close to 0·475 (compact arrangement of particles in configuration A, *i.e.* $L = 2R$) the configuration A, and, of course also B, yields V_0/u practically identical with the empirical value (curve *1*). It is also apparent from the figure that all our configurations predict a correct trend for the superficial fluid velocity and in the region $\varepsilon = 0{\cdot}45$ to 0·68 provide a better fit than the so far published correlations. For the limiting porosity $\varepsilon \to 1$ our configurations, same as other models, predict lower values of the superficial velocity with respect to the experiment.

CONCLUSION

The true course of the fluidized bed expansion is in all cases predicted better by our model than by other theoretically based ones. In spite of this, the most reliable approach at present is through empirical equations, as noted *e.g.* by Todes[2]. In case of fluidization by gases one has to anticipate eventual presence of induction of an electrostatic charge. In such cases the safest way of determining the superficial velocity of fluidization is by means of an experiment.

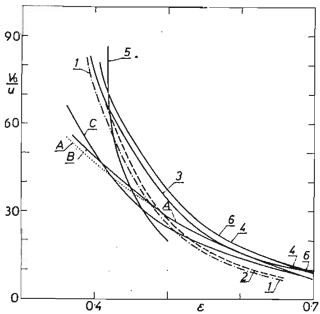

FIG. 2

V_0/u as a Function of Porosity

A, B, C — models formulated in this work, *1* $\varepsilon^{-4.65}$, *2* $\varepsilon^{-4.75}$, *3* Ergun equation[8], *4* Happel equation[7], *5* Brinkman equation[4], *6* Kozeny–Carman equation in Todes' modification[2,5,6].

From the foregoing it may be concluded that in spite of the relative simplicity of the model, the results are quite promising. On the contrary, the model does not agree with the axioms of the hydrodynamics in that it does not yield continuous derivatives of velocity (*i.e.* of the stress tensor) throughout the cross section for the configurations *A*, *B* and *C*. The reason for the discontinuities is in the formal application of Eq. (*1*), strictly valid for an infinite system to a confined space. On the other hand, the substantial physical simplification of the problem permitted its easy solution. Another controversial feature is the validity of Eq. (*1*) in the proximity of the points of contact of particles. From the analogy of the flow of fluids in conduits of non-circular cross section it can be inferred on the instability of the flow[9] in these regions (Taylor's waves). The question to which extent this effect may influence the superficial velocity of fluidization cannot be answered at present.

APPENDIX

The Brinkman equation[4]:

$$u/V_0 = [1 + 2\cdot5(1 - \varepsilon)]\,[1 + 3/4(1 - \varepsilon)\,(1 - \sqrt{(8/(1 - \varepsilon) - 3)}]\,.$$

The Kozeny–Carman equation[5,6] in Todes'modification:

$$u/V_0 = \varepsilon^3/[\varepsilon^3 + 9(1 - \varepsilon)]\,.$$

The Ergun equation[8] (confined to a single term only)

$$u/V_0 = 0\cdot12\varepsilon^3/(1 - e).$$

The Happel equation[7]:

$$u/V_0 = \{1 - 1\cdot5(1 - \varepsilon)^{1/3} + 1\cdot5(1 - \varepsilon)^{5/3} - (1 - \varepsilon)^2\}/[1 + 0\cdot4(1 - \varepsilon)^{5/3}]\,.$$

LIST OF SYMBOLS

L distance of the centers of neighbouring particles in the bed
R radius of particles
r distance from particle's center
S_1 free area of cross section
S cross sectional area in the elementary cell of the bed in the equatorial plane
u superficial velocity of fluidization
u_z projection of local velocity into z direction
V_0 terminal velocity of an isolated particle
ε porosity
φ angle

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